Prediction of shear strength of FRP-reinforced concrete beams without stirrups based on genetic programming

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A B S T R A C T

The use of fibre reinforced polymer (FRP) bars to reinforce concrete structures has received a great deal of attention in recent years due to their excellent corrosion resistance, high tensile strength, and good non-magnetization properties. Due to the relatively low modulus of elasticity of FRP bars, concrete members reinforced longitudinally with FRP bars experience reduced shear strength compared to the shear strength of those reinforced with the same amounts of steel reinforcement. This paper presents a simple yet improved model to calculate the concrete shear strength of FRP-reinforced concrete slender beams (a/d > 2.5) without stirrups based on the gene expression programming (GEP) approach. The model produced by GEP is constructed directly from a set of experimental results available in the literature. The results of training, testing and validation sets of the model are compared with experimental results. All of the results show that GEP is a strong technique for the prediction of the shear capacity of FRP-reinforced concrete beams without stirrups. The performance of the GEP model is also compared to that of four commonly used shear design provisions for FRP-reinforced concrete beams. The proposed model produced by GEP provides the most accurate results in calculating the concrete shear strength of FRP-reinforced concrete beams among existing shear equations provided by current provisions. A parametric study is also carried out to evaluate the ability of the proposed GEP model and current shear design guidelines to quantitatively account for the effects of basic shear design parameters on the shear strength of FRP-reinforced concrete beams.

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1. Introduction

In recent years, fibre reinforced polymer (FRP) bars have been adopted as a potential solution to the corrosion problems in concrete structures. In addition to their excellent non-corrosive characteristics, FRP reinforcements have high strength-to-weight ratio, good fatigue properties and electro-magnetic resistance [1,2]. There are fundamental differences between the steel and FRP reinforcements: the latter has a lower modulus of elasticity and linear stress–strain diagram up to rupture with no discernible yield point and different bond strength according to the type of FRP product. Due to the relatively low modulus of elasticity of FRP bars, concrete members reinforced longitudinally with FRP bars experience reduced shear strength compared to the shear strength of those reinforced with the same amounts of steel reinforcement. This fact is supported by the findings from the experimental investigations on FRP-reinforced concrete beams [3–5].

The applied shear stresses in a cracked reinforced concrete member without transverse reinforcement are resisted by various shear mechanisms. The Joint ASCE-ACI Committee 445 [6] assessed that the quantity of concrete shear strength $V_c$ can be considered as a combination of five mechanisms activated after the formation of diagonal cracks: (1) shear stresses in uncracked compressed concrete; (2) aggregate interlock; (3) dowel action of the longitudinal reinforcing bars; (4) arch action; and (5) residual tensile stresses transmitted directly across the cracks. The contribution of the uncracked concrete in reinforced concrete members depends mainly on the concrete strength, $f'_c$, and on the depth of the uncracked zone, which is function of the longitudinal reinforcement properties. Aggregate interlock results from the resistance to relative slip between two rough interlocking surfaces of the crack, much like frictional resistance. The dowel action refers to the shear force resisting transverse displacement between two parts of a structural element split by a crack that is bridged by the reinforcement. Arching action occurs in deep members or in members in which the shear span-to-depth ratio (a/d) is less than 2.5. This is not a shear transfer mechanism in the sense that it does not transmit a tangential force to a nearby parallel plane, but permits the transfer of a vertical concentrated force to a reaction, thereby reducing the contribution of the other types of shear transfer. The basic explanation of residual tensile stresses is that when concrete first cracks, a clean break does not occur. The residual tension in cracked
concrete has been found to be present for crack widths smaller than 0.15 mm [5,7].

Due to the relatively low modulus of elasticity of FRP composite material, concrete members reinforced with FRP bars will develop wider and deeper cracks than members reinforced with steel. Deeper cracks reduce the contribution to shear strength from the uncracked concrete due to the lower depth of concrete in compression. Wider cracks in turn reduce the contributions from aggregate interlock and residual tensile stresses. Additionally, due to the relatively small transverse strength of FRP bars and relatively wider cracks, the contribution of dowel action can be very small compared to that of steel. Finally, the overall shear capacity of concrete members reinforced with FRP bars as flexural reinforcement is lower than that of concrete members reinforced with steel bars [5].

Previous studies [4,8–10] concluded that current shear design guidelines are very conservative in calculating the shear capacity of FRP-reinforced concrete beams. Consequently, the excessive amount of FRP needed to resist shear could be both costly and likely to create reinforcement congestion problems [8]. Accordingly, the purpose of this paper is to develop a simple yet accurate model for predicting the shear strength of FRP-reinforced concrete slender beams \((a/d > 2.5)\) without stirrups. GEP approach is also used to build empirical model. For building the model, shear capacity results of 104 specimens used in training, testing and validation sets for GEP model were obtained from the literature. Six main parameters that affect the shear strength of FRP-reinforced concrete members were selected for input variables. In the sets of variables for GEP model were obtained from the literature. Six main parameters that affect the shear strength of FRP-reinforced concrete members were selected for input variables. In the sets of variables for GEP model were obtained from the literature. Six main parameters that affect the shear strength of FRP-reinforced concrete members were selected for input variables. In the sets of variables for GEP model were obtained from the literature. Six main parameters that affect the shear strength of FRP-reinforced concrete members were selected for input variables. In the sets of variables for GEP model were obtained from the literature. Six main parameters that affect the shear strength of FRP-reinforced concrete members were selected for input variables. In the sets of variables for GEP model were obtained from the literature. Six main parameters that affect the shear strength of FRP-reinforced concrete members were selected for input variables.

### 2. Review of current design provisions

Due to the rapid increase of using FRP materials as reinforcement for concrete structures, there are international efforts to develop design guidelines. These efforts have resulted in the publishing of several codes and design guidelines. Most of the shear design provisions incorporated in these codes and guides on shear capacity of FRP-reinforced concrete beams have focused on modifying existing shear design equations for steel-reinforced concrete beams to account for the substantial differences between FRP and steel reinforcement. These provisions are generally based on the parallel truss model with 45° constant inclination diagonal shear cracks. This model identifies the shear strength of a reinforced concrete flexural member as the sum of the shear capacity of the concrete component \(V_{cf}\) and the shear reinforcement component \(V_s\). In this paper, the concrete shear strength component \(V_{cf}\) of members longitudinally reinforced with FRP bars as recommended by ACI 440, ISIS Canada, CSA S806, and JSCE are reviewed and they are listed in Table 1. Note that all strength reduction factors used in the equations listed in the table for design purposes are set equal to one for comparison.

### 3. Genetic programming approach

Genetic programming (GP) is proposed by Koza [16]. It is a generalization of genetic algorithms (GAs) [17]. The most general form of a solution to a computer-modelled problem is a computer program. GP takes cognizance of this and attempts to use computer programs as its data representation. Similarly to GA, GP needs only the problem to be defined. Then, the program searches for a solution in a problem-independent manner [16–18].

GP breeds computer programs to solve problems by executing the following three steps:

1. (1) Generate an initial population of random compositions of the functions and terminals of the problem.

2. (2) Iteratively perform the following substeps until the termination criterion has been satisfied:
   - (A) Execute each program in the population and assign it a fitness value using the fitness measure.

### Table 1

Shear design equations for FRP-reinforced concrete beams without stirrups.

<table>
<thead>
<tr>
<th>Code</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACI 440-03</td>
<td>(V_d = \frac{a b_c C}{600} ) (V_e \leq V_c)</td>
</tr>
<tr>
<td>ACI 440-06</td>
<td>(V_d = \sqrt{\frac{a b_c C}{600}} ) (C - Kd)</td>
</tr>
<tr>
<td>CSA S806-02</td>
<td>(V_d = 0.036 b_d \sqrt{\frac{V_c}{f_c}}) (1/3) (0.6 b_d \sqrt{f_c} \leq V_d \leq 0.2 b_d \sqrt{f_c}) (d \leq 300) mm</td>
</tr>
<tr>
<td>JSCE-97</td>
<td>(V_d = \frac{0.1 b_d \sqrt{f_c}}{23} \leq V_c \leq 1.5 \beta_d \leq 1.5)</td>
</tr>
<tr>
<td>ISIS Canada-01</td>
<td>(V_d = 0.2 b_d \sqrt{f_c} ) (d \leq 300) mm</td>
</tr>
</tbody>
</table>

Note: \(f_c\) = compressive strength of concrete, \(b_c\) and \(d\) = beam's width and effective width, respectively, \(\rho_l\) = longitudinal reinforcement ratio; \(E_s\), \(E_c\), and \(E_f\) = modulus of elasticity of concrete, steel and FRP longitudinal bars, respectively; \(M_0\) and \(V_0\) = moment and shear force at critical section, respectively.
(B) Create a new population of computer programs by applying the following operations. The operations are applied to computer program(s) chosen from the population with a probability based on fitness.

(i) Reproduction: Copy an existing program to the new population.

(ii) Crossover: Create new offspring program(s) for the new population by recombining randomly chosen parts of two existing programs, as seen in Fig. 1.

(iii) Mutation. Create one new offspring program for the new population by randomly mutating a randomly chosen part of one existing program, as seen in Fig. 2.

(3) The program that is identified by the method of result designation (e.g., the best-so-far individual) is designated as the result of the genetic programming system for the run. This result may be a solution (or approximate solution) to the problem [19,20].

3.1. Gene expression programming approach

Ferreira [21] suggested a new algorithm based on GA and GP. This algorithm develops a computer program encoded in linear chromosomes of fixed length. The GEP, which performs the symbolic regression using the most of the genetic operators of GA, fundamentally aims to find a mathematical function principal using a set of data presented [22,23].

The basic GEP algorithm is depicted in Fig. 3. To develop a GEP model, five components; the function set, terminal set, fitness function, control parameters and stop condition are required. After the problem is encoded for candidate solution and the fitness function is specified, the algorithm randomly creates an initial population of viable individuals (chromosomes) and then converts the each chromosome into an expression tree corresponding to a mathematical expression. Afterwards the predicted target is compared with the actual one and the fitness score for each

![Fig. 1. Example of genetic programming crossover.](image1)

![Fig. 2. Example of genetic programming mutation.](image2)
chromosome is determined. If it is sufficiently good, the algorithm stops. Otherwise, some of the chromosomes are selected using roulette wheel sampling and then mutated to obtain the new generations. This closed loop is continued until desired fitness score is achieved and then the chromosomes are decoded for the best solution of the problem [24,25].

GEP has two main elements such as the chromosomes and the expression trees (ETs). The chromosomes may be consisted of one or more genes which represents a mathematical expression. The mathematical code of a gene is expressed in two different languages called Karva Language [26,27]: such as the language of the genes and the language of the ETs. The genes have two main parts addressed as the head and the tail. The head includes some mathematical operators, variables and constants (+, –, *, /, √, sin, cos, 1, a, b, c) which are used to encode a mathematical expression. The tail just includes variables and constants (1, a, b, c) named as terminal symbols. Additional symbols are used if the terminal symbols in the head are inadequate to define a mathematical expression. A simple chromosome as linear string with one gene is encoded in Fig. 4. Its ET and the corresponding mathematical equation are also shown in same figure. The translation of ET to Karva Language is done by beginning to read from left to right in the top line of the tree and from top to bottom. The sequences of genes used in this method are similar to sequences of biological genes and have coding and non-coding parts. On the other hand more complex mathematical equations are defined by more than one chromosome referred to multigenic chromosomes. Joining of the genes is done by linking function such as addition, subtraction, multiplication, or division [23,25].

3.2. Experimental database

In this study, shear strength results of 104 specimens given in Table 2 were collected from published literature [4,5,8,29–40]. The specimens included 91 beams and 13 one-way slabs; all were simply supported and were tested either in three-point or four-point bending. These specimens included two specimens reinforced with aramid FRP bars, 36 specimens reinforced with carbon FRP bars, and 66 specimens reinforced with glass FRP bars. All specimens had no transverse reinforcement and exhibited shear failure. The concrete compressive strength, \( f_{0c} \), of the test specimens ranged between 24.1 and 81.4 MPa. The reinforcement ratio, \( q_f \), ranged between 0.25 and 3.02%; the shear span-to-depth ratio, \( a/d \), ranged between 2.53 and 6.5; and the effective depth, \( d \), ranged between 141 and 360 mm. Table 2 shows relevant details on the specimens.

3.3. Gene expression programming model

In the present study, six main parameters that affect the shear strength of FRP-reinforced concrete members without stirrups were selected for input variables. In training and testing of the GEP model, \( f_{0c} \), \( b_w \), \( d \), \( a/d \), \( q_f \) and \( E_f/E_s \) were entered as input variables, while \( V_{cf} \) value was used as output variable. Among 104 experimental sets taken from the literature, 56 sets were randomly chosen as a training set for the GEP modeling and 28 sets were used as testing the generalization capacity of the proposed model.
### Table 2
Training, testing and validation database for FRP-reinforced concrete members.

<table>
<thead>
<tr>
<th>References</th>
<th>Beam</th>
<th>$f_c$ (MPa)</th>
<th>$b_w$ (mm)</th>
<th>$d$ (mm)</th>
<th>$a$ (mm)</th>
<th>Reinforcement</th>
<th>$\eta_f$ (%)</th>
<th>$E_f$ (GPa)</th>
<th>$V_{exp}$ (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>[4]</td>
<td>1FRPa</td>
<td>36.3</td>
<td>229</td>
<td>225</td>
<td>914</td>
<td>1.11</td>
<td>40.3</td>
<td>39.1</td>
<td></td>
</tr>
<tr>
<td>[4]</td>
<td>1FRPb</td>
<td>36.3</td>
<td>229</td>
<td>225</td>
<td>914</td>
<td>1.11</td>
<td>40.3</td>
<td>38.5</td>
<td></td>
</tr>
<tr>
<td>[4]</td>
<td>1FRPc</td>
<td>36.3</td>
<td>229</td>
<td>225</td>
<td>914</td>
<td>1.11</td>
<td>40.3</td>
<td>36.8</td>
<td></td>
</tr>
<tr>
<td>[4]</td>
<td>2FRPa</td>
<td>36.3</td>
<td>178</td>
<td>225</td>
<td>914</td>
<td>1.42</td>
<td>40.3</td>
<td>28.1</td>
<td></td>
</tr>
<tr>
<td>[4]</td>
<td>2FRPb</td>
<td>36.3</td>
<td>178</td>
<td>225</td>
<td>914</td>
<td>1.42</td>
<td>40.3</td>
<td>35</td>
<td></td>
</tr>
<tr>
<td>[4]</td>
<td>2FRPc</td>
<td>36.3</td>
<td>229</td>
<td>225</td>
<td>914</td>
<td>1.66</td>
<td>40.3</td>
<td>40</td>
<td></td>
</tr>
<tr>
<td>[4]</td>
<td>3FRPa</td>
<td>36.3</td>
<td>229</td>
<td>225</td>
<td>914</td>
<td>1.66</td>
<td>40.3</td>
<td>48.6</td>
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<tr>
<td>[4]</td>
<td>3FRPb</td>
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<td>914</td>
<td>1.66</td>
<td>40.3</td>
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<tr>
<td>[4]</td>
<td>3FRPc</td>
<td>36.3</td>
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<td>225</td>
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<td>1.66</td>
<td>40.3</td>
<td>43.8</td>
<td></td>
</tr>
<tr>
<td>[4]</td>
<td>4FRPa</td>
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<td>225</td>
<td>914</td>
<td>1.81</td>
<td>40.3</td>
<td>45.9</td>
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<tr>
<td>[4]</td>
<td>4FRPb</td>
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<td>40.3</td>
<td>46.3</td>
<td></td>
</tr>
<tr>
<td>[4]</td>
<td>4FRPc</td>
<td>36.3</td>
<td>229</td>
<td>225</td>
<td>914</td>
<td>1.81</td>
<td>40.3</td>
<td>42.5</td>
<td></td>
</tr>
</tbody>
</table>

(continued on next page)
and solution is less than or equal to 0.01, then the precision is equal to zero, and the set of terminals 

$$T_{te}$$

and the set of functions 

$$F$$

are chosen, namely, 

$$C_{i}^{(j)}$$

were used.

basic arithmetic operators 

$$+ \, \, / \, \, *)$$

and some basic mathematical functions 

$$\{ \sqrt{.} \, \, \min \, \, \max \, \, \exp \}$$

were used.

$$\nu = \{ b_0, b_0, d, q/d, \rho_c, E_f/E_{0} \}$$

and two basic arithmetic operators 

$$(+ , \, /)$$

and some basic mathematical functions (Cubic Root, Mul3) were used.

$$\nu_3 = \{ b_0, b_0, d, a/d, \rho_c, E_f\}$$

and two basic arithmetic operators 

$$(+ , \, /)$$

and some basic mathematical functions (Cubic Root, Mul3) were used.

Another major step is to choose the chromosomesal tree, i.e., the length of the head and the number of genes. The GEP approach model initially used single gene and two lengths of heads, and increased the number of genes and heads, one after another during each run, and monitored the training and testing sets performance of the model. In the present study after several trials, length of the head, 

$$h = 6$$

and two genes were found to give the best results. The sub-ETs (genes) were linked by multiplication.

Finally, a combination of all genetic operators (mutation, transposition and crossover) is utilized as set of genetic operators. Parameters of the training of the GEP approach model are given in Table 4. Chromosome 32 was observed to be the best of generation individuals predicting 

$$V_{cf}$$

having fitness of 837. Explicit formulation based on the GEP approach model for 

$$V_{cf}$$

is obtained by

$$V_{cf} = b_0 \cdot d \left( \left( \frac{d}{d_f} \right)^{1/3} \frac{E_f}{E_{0}} \right)^{1/3} \left( c_0/c_2 \right)$$

The expression tree of formulation for the 

$$V_{cf}$$

is also shown also in Fig. 5 where 

$$d_0, d_1, d_2, d_3, d_4$$

and 

$$d_5$$

refer to 

$$f_j$$

and 

$$\rho_c$$

and 

$$E_f/E_{0}$$

respectively. The constants in the formulation are; 

$$c_0 = 7.696$$

and 

$$c_2 = 7.718$$.

### 4. Results and discussion

In the present study, all strength reduction factors used in the shear equations for design purposes are set equal to one for comparison. Of the 104 experimental data were utilized for training, testing and validation sets for GEP model. The validation data are unfamiliar to the model and were not included in its development. All of the results obtained from experimental studies and predicted by using the training, testing and validation results of the model are given in Fig. 6. As seen in Fig. 6, the results obtained from the model are compared to the experimental results for training.
testing and validation sets, respectively. The training set results prove that the proposed model has impressively well learned the non-linear relationship between the input and the output variables with good correlation. Comparing the GEP model predictions with the experimental results for the testing and validation stages demonstrates a high generalization capacity of the proposed model.

The performance of Eq. (2) and equations provided by current shear design guidelines and recommendations in calculating the shear strength of FRP-reinforced concrete beams without stirrups has been evaluated using the training, testing and validation database described earlier, based on both the ratio of experimentally measured to analytically calculated shear strength $V_{\text{cal}}/V_{\text{exp}}$ (the ratio of the shear resistance attained experimentally to the corresponding analytical value), and the average absolute error AAE calculated using Eq. (3).

$$\text{AAE} = \frac{1}{n} \sum \frac{|V_{\text{exp}} - V_{\text{cal}}|}{V_{\text{exp}}} \times 100$$ (3)

Table 5 reports the average and standard deviation (SD) for $V_{\text{exp}}/V_{\text{cal}}$, and the AAE of all shear design equations. It can be seen that the proposed model has the lowest AAE of 13.4% compared to 42.3% for ACI-06, 21.3% for CSA-02, 22.8% for JSCE-97, and 31.6% for ISIS-01. The GEP model also provides the least ratio of experimentally measured to analytically calculated shear strength ($V_{\text{exp}}/V_{\text{cal}}$) value. Thus, the proposed model appears to be more accurate and reliable for predicting the concrete shear strength for flexural members longitudinally reinforced with FRP bars.

Fig. 7 also shows the performance of the model produced by GEP and those provided by commonly used shear design standards and recommendations. The ratio of experimentally measured to analytically calculated shear strength, $V_{\text{exp}}/V_{\text{cal}}$ for all beams is shown in the figure. It is clear that the shear design equation provided by the latest version of ACI shear design guidelines for FRP-reinforced beams (ACI 440-06) shows an improved prediction over ACI 440-03, and is better estimated the shear capacity of FRP-reinforced concrete beams with an average $V_{\text{exp}}/V_{\text{cal}}$ of 1.79 (3.68 for ACI 440-03). Shear design equations of CSA S806-02, JSCE-97, and ISIS Canada-01 provides better results than that of ACI 440-06. On the other hand, GEP model gives the most accurate results for the shear strength of FRP-reinforced concrete beams with an average $V_{\text{exp}}/V_{\text{cal}}$ equal to 1.03. The proposed model also includes the most shear design parameters that influence the shear capacity of FRP-reinforced concrete beams as the shear design equation of CSA.

The effect of axial stiffness of FRP bars on the shear capacity FRP-reinforced concrete beams is assumed to be to the magnitude of $(E_f/E_s)^{1/2}$ by ISIS, whereas such an effect is considered to be $(E_f/E_s)^{1/3}$ by Eq. (2), JSE and CSA guidelines. Moreover, the ISIS method does not take into account the contribution of other common shear design parameters on $V_{\text{cal}}$, such as the shear span-to-depth ratio, $a/d$, and the longitudinal reinforcement ratio $\rho_l$. This could explain why the ISIS method gives the relatively higher value of SD.

Fig. 8 through Fig. 10 also present the experimental-to-calculated shear strength versus compressive strength, axial stiffness of reinforcing bars and shear span-to-depth ratio. From the Figs. 8–10, it is evident that the level of accuracy of the shear strength predicted by the GEP model equation seems to be consistent with the varying $a/d$ ratio, compressive strength ($f_c$) and axial stiffness of reinforcing bars $(E_f/E_s)$.

4.1 Parametric study on effect of basic shear design parameters

4.1.1 Effect of longitudinal reinforcement ratio on shear strength

In the present study, a sensitivity analysis has been conducted using the GEP model to investigate the effect of longitudinal reinforcement ratio on the shear strength of FRP-reinforced concrete.
The shear strength of a set of beams having geometrical and mechanical properties similar to those of beams randomly selected from the database [8] have been also calculated for different amounts of longitudinal reinforcement ratio using the shear design methods considered herein. Fig. 11 presents the effect of $\rho_f$ on the shear strength of reinforced concrete beams. It is shown that all methods, including the GEP model take into account similar influence for the effect of $\rho_f$ on shear strength. However, a linear relationship is assumed by ACI 440-03 for such an effect, as opposed to a non-linear effect for the other shear design methods.
4.2. Effect of concrete compressive strength on shear strength

To investigate the ability of shear design guidelines to quantitatively consider the effect of $f'_c$ on shear strength of FRP-reinforced concrete beams, a set of six beams similar to a beam randomly selected from the database and tested by Razaqpur et al. [8] is considered. Fig. 12 shows the variation in shear strength of FRP-reinforced concrete beams with variable concrete compressive strength. The figure illustrates the effect of $f'_c$ as estimated by the GEP model and various shear provisions considered in this study. It is shown that all shear design methods consider the effect of $f'_c$, but they vary in the magnitude of such an effect. The shear design method provided by ACI 440-03 assumes that the shear strength of FRP-reinforced concrete beams decreases as $f'_c$ increases, whereas all other methods, including the GEP model, assume that the shear strength of FRP-reinforced concrete beams increases with an increase of concrete compressive strength (Fig. 12).

4.3. Effect of shear span-to-depth ratio on shear strength

Fig. 13 shows the relationship between shear span-to-depth ratio ($a/d$) and the shear strength of a set of beams calculated using the GEP model and shear design methods considered herein. The figure also includes the experimental shear strength of a similar beam measured by El-Sayed et al. [30]. While ACI 440 and JSCE shear design provisions do not consider the effect of $a/d$ on the shear strength of reinforced concrete beams, CSA S806 and GEP model responses exhibit a slight influence of $a/d$ on the shear strength of FRP-reinforced concrete beams without stirrups.

**Fig. 13.** Effect of the shear span-to-depth ratio ($a/d$) on the shear strength of FRP-reinforced concrete beams without stirrups.

![Graph showing the relationship between shear span-to-depth ratio ($a/d$) and shear strength of FRP-reinforced concrete beams without stirrups.](image_url)

5. Conclusions

This study reports an efficient approach for the formulation of shear strength of FRP-reinforced concrete beams using GEP. An empirical model to predict the shear strength of FRP-reinforced concrete beams without web reinforcement has been obtained by GEP approach. Experimental results are used to build and validate the model. Good agreement between the model predictions and experiments has been achieved. The values of the average absolute error AAE, and the average and standard deviation for $V_{cal}/V_{exp}$ have shown this situation.

The GEP model equation also gives good predictions for the shear strength of FRP-reinforced concrete beams with the varying $a/d$ ratio, compressive strength of concrete and axial stiffness of reinforcing bars.

Shear provisions of ACI 440 are highly conservative in estimating the shear strength of FRP-R/C beams without shear reinforcement. All other shear provisions considered in this study also gives conservative results for such beams even without applying reduction factors.

The proposed model has been compared to the current guidelines and provisions. More accurate and consistent predictions have been obtained using the model produced by GEP.

Shear design equation produced by GEP model accounts for the effect of the axial stiffness of FRP bars on shear capacity of FRP-reinforced concrete beams as a cubic root function of $E_f/E_s$. It provides the most accurate results in calculating $V_f$.

The proposed model is so simple that they can be used by anyone not necessarily being familiar with GEP. The model also gives a practical way for the prediction of concrete shear strength of beams reinforced with FRP bars to obtain accurate results, and encourages use of GEP in other aspects of civil engineering studies.

References


